Reductions, Self-Similarity, and Recursion

Relations between problems

Notes for CSC 100 - The Beauty and Joy of Computing
The University of North Carolina at Greensboro

Reminders

Blown to Bits: Chapter 4 discussion over the next week

Homework 3:

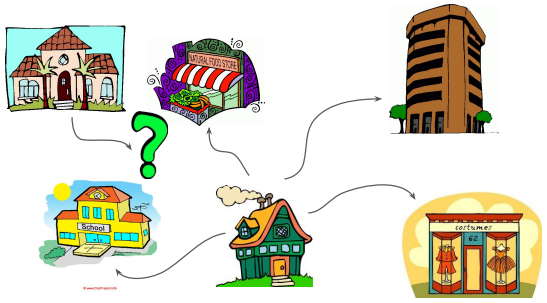
- Read assignment carefully to make sure you understand it
- Get started! Goal: At least watch the video by Friday

Projects: Think about ideas, talk to other students, …

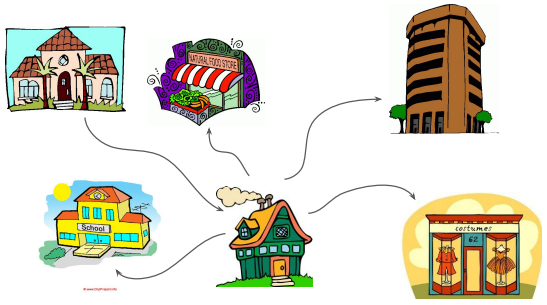
We will discuss more about project ideas / teams on Wednesday

Getting to places from my house...

Now I buy a new house!



Get anywhere by first going to old house



Things to notice...

What I want to do...

I can *go anywhere from my new house* by

1. Going to my old house
2. Going to my destination from there

What I know how to do...

Things to notice...

What I want to do...

I can *go anywhere from my new house* by

1. Going to my old house
2. Going to my destination from there

What I know how to do...

Terminology: I have *reduced the problem of traveling from my new house to the problem of traveling from my old house*.

Important points:

- Solution is easy to produce (often easier than direct solution)
- Solution is easy and compact to describe (especially with abstraction!)
- Solution may *not* be the most efficient to execute

Things to notice...

What I want to do...

I can *go anywhere from my new house* by

1. Going to my old house
2. Going to my destination from there

What I know how to do...

Question: Is a reduction a property of problems or algorithms?

Things to notice...

Problem

I can *go anywhere from my new house* by

1. Going to my old house
2. *Going to my destination from there*

Problem

Reductions are between *problems*

- The reduction operation is an algorithm
- Abstraction: We don't care how the "known algorithm" works!

The Basics

A *reduction* is using the solution of one problem (problem A) to solve another problem (problem B).

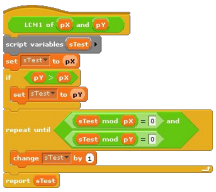
We say "problem B is reduced to problem A".

Reductions are a fundamental "big idea" in computer science

- Lots of types of reductions - you could spend a lifetime studying this!
- Our reductions use a small amount of work in addition to a constant number of calls to problem A.
  - As a result, can say problem B is not much harder than problem A
  - True even if we don't know the most efficient way to solve problem A!

An example from Mathematics

To find least common multiple (LCM):



An example from Mathematics

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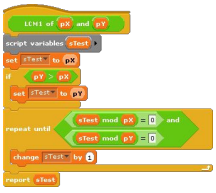
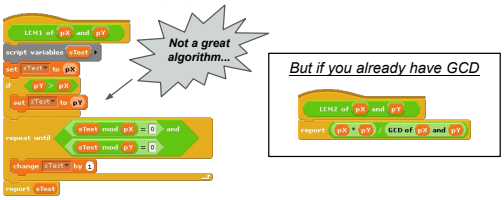


But if you already have GCD

What have we done? We have *reduced the problem of computing LCM to the problem of computing GCD*.

An example from Mathematics

To find least common multiple (LCM):



But if you already have GCD

What have we done? We have *reduced the problem of computing LCM to the problem of computing GCD*.

So: LCM is no harder computationally than GCD. And remember... Euclid's algorithm is a very efficient GCD algorithm!

Similarity and Self-Similarity

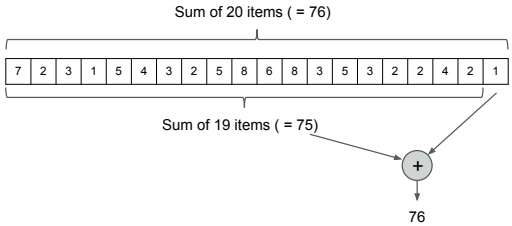
Reducing LCM to GCD identifies similarities between the two problems.

Many problems are structured so that solutions are "self-similar" - large solutions contain solutions to smaller versions of the same problem!

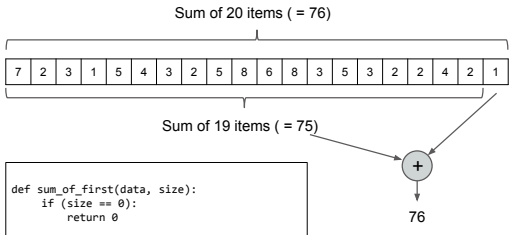
Example: Think about adding up the numbers in an *n*-element list. Adding up the first *n*-1 elements is a smaller version of the same problem!

An algorithm can solve a large problem by breaking it down to smaller versions of the *same* problem - this is called *recursion*.

Example: Adding up a list



Example: Adding up a list




```
def sum_of_first(data, size):
    if (size == 0):
        return 0

    subProblem = sum_of_first(data, size-1)
    return subProblem + data[size-1]
```

Breaking it down

```
def sum_of_first(data, size):
    if (size == 0):
        return 0

    subProblem = sum_of_first(data, size-1)
    return subProblem + data[size-1]
```

Base case: Handling smallest case directly

Recursive case: Solving a smaller version of the same problem.

Constant amount of work to use answer from subproblem to compute answer to overall problem.

Breaking it down

Workhorse Function

```
def sum_of_first(data, size):
    if (size == 0):
        return 0

    subProblem = sum_of_first(data, size-1)
    return subProblem + data[size-1]
```

Base case: Handling smallest case directly

Recursive case: Solving a smaller version of the same problem.

Constant amount of work to use answer from subproblem to compute answer to overall problem.

Driver Function

```
def sum_of(data):
    return sum_of_first(data, len(data))
```

Driver function: sets up first call to recursion

Another example: Sorting

"Selection sort" from algorithms lab:

```
def sort(data):
    for left in range(len(data), 1, -1):
        maxPos = max_pos_from_first(data, left)
        swap(data, maxPos, left-1)
```

Another example: Sorting

"Selection sort" from algorithms lab:

```
def sort(data):
    for left in range(len(data), 1, -1):
        maxPos = max_pos_from_first(data, left)
        swap(data, maxPos, left-1)
```

Recursive version:

```
def recursive_sort(data, size):
    if (size > 1):
        maxPos = max_pos_from_first(data, size)
        swap(data, maxPos, size-1)
        recursive_sort(data, size-1)
```

Base case: One item - nothing to do!

Setting up recursion: Swap max item to last position

Recursion: Sort all the rest

Note the elegance of the recursive description: “If there’s something to sort, put the largest item at the end and then sort the rest.”

Summary

Finding relations between problems can simplify solutions:

- Sometimes relations between different problems (reductions)
- Sometimes relation to smaller version of the same problem (recursion)

What you should know:

- Recognize reductions and recursion
- Understand the basic principles

We will explore this more in a lab!