CSC 580
Cryptography and Computer Security

Cryptographic Hash Functions
(Chapter 11)

March 28, 2017

Overview

Today:

- Review Homework 8 solutions
- Discuss cryptographic hash functions

Next:

- Study for quiz on Homework 8
- Read Chapter 12.1-12.5

Hash Function Basics and Terminology

General Definition: A **hash function** maps a large domain into a small, fixed-size range. Domain often generalized to all binary strings.

$$H: \{0,1\}^* \rightarrow R$$

↖ *Fixed size range*

Use in data structures: *R* is set of hash table indices.

Important properties:

- Efficient to compute
- Uniform distribution ("apparently random")

If $H(x)=h$, then we say "x is a **preimage** of h"

If $x \neq y$, but $H(x) = H(y)$, then the pair (x,y) is a **collision**

Question: Do all hash functions have collisions?

Cryptographic Hash Functions

Cryptographic hash functions map to fixed-length bit-vectors, sometimes called **message digests**.

$$H: \{0,1\}^* \rightarrow \{0,1\}^n$$

For cryptographic applications, need one or more of these properties:

- **Preimage resistance**: Given *h*, it's infeasible to find *x* such that *H*(*x*)=*h*
  - Also called the "*one-way property*"
- **Second preimage resistance**: Given *x*, it's infeasible to find *y* ≠ *x* such that *H(x)*=*H*(*y*)
  - Also called "*weak collision resistance*"
- **Collision resistance**: It's infeasible to find any two *x* and *y* such that *x* ≠ *y* and *H*(*x*)=*H*(*y*)
  - Also called "*strong collision resistance*"

The SHA Family of Algorithms

SHA is the "Standard Hash Algorithm"

Table 11.3 from the textbook:

Algorithm	Message Size	Block Size	Word Size	Message Digest Size
SHA-1	$< 2^{64}$	512	32	160
SHA-224	$< 2^{64}$	512	32	224
SHA-256	$< 2^{64}$	512	32	256
SHA-384	$< 2^{128}$	1024	64	384
SHA-512	$< 2^{128}$	1024	64	512
SHA-512/224	$< 2^{128}$	1024	64	224
SHA-512/256	$< 2^{128}$	1024	64	256

Note: MD5 is an older algorithm with a 128-bit digest - don't use MD5 or SHA-1.

Thinking about Collisions

If hashing *b*-bit inputs to *n*-bit digests, how many preimages per digest?

- Worst case?
- On average?

[redacted]

Thinking about Collisions

If hashing *b*-bit inputs to *n*-bit digests, how many preimages per digest?

- Worst case ("at least *c* preimages for some digest…")?
- On average?

For worst case:

If there are *m* items to be put into *n* bins, then one bin must contain at least $\lceil m/n \rceil$ items (generalization of the pigeonhole principle).

2^b preimages "placed in" 2^n preimage bins
➔ One digest must have at least $\lceil 2^b/2^n \rceil = 2^{b-n}$ preimages

Thinking about Collisions

If hashing *b*-bit inputs to *n*-bit digests, how many preimages per digest?

- Worst case?
- On average?

For average case:

Let p_h be the number of preimages for hash value (digest) h.

Since each of the 2^b preimages is the preimage to exactly one digest,

$$\sum_h p_h = 2^b.$$

The average number of preimages for any digest is therefore

$$\frac{\sum_h p_h}{2^n} = \frac{2^b}{2^n} = 2^{b-n}$$

Thinking about Collisions
Some real numbers

Using SHA-1 to hash 256-bit (32-byte) inputs:
➔ A digest has on average $2^{256-160} = 2^{96}$ different preimages

Bottom line: Lots and lots and lots and lots of collisions!

Looking for 2^{96} needles in a size 2^{256} haystack still is hard...

MD5 was introduced in 1992

- Not a single collision found until 2004
- Now finding collisions in MD5 is fairly routine

SHA-1 was introduced in 1995

- Not a single collision found until… Feb 23, 2017
- Recommendations to not use since 2010
- Don't use any more!

Brute Force Attacks
On Preimage and Second Preimage Resistance

Brute force attack to find a preimage:

```
find-preimage(h)   // h is n bits
   repeat
      x ← random input
   until H(x) = h
```

If *H* is uniformly distributed: prob $1/2^n$ of finding preimage each time

This is a Bernoulli trial with success probability $1/2^n$

- ➔ Repeat until success gives a geometric distribution
- ➔ Expected number of trials is 2^n

Question: What about a brute force attack to find a second preimage?

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Question: What about a brute force attack to find a second preimage?

Answer: Same analysis… expected number of test hashes is 2^n

Brute Force Attacks
On Collision Resistance

Free to match up *any* two preimages for a collision, so:

```
S ← {}
while true:
   x ← random input
   if a pair (y,H(x)) is in S with y ≠ x then
      return (x,y)
   Add (x,H(x)) to S
```

Looking for any duplicate pair is the "Birthday Problem"

- ➔ Picking randomly from *m* items
- ➔ Expect a duplicate after ≈ $\sqrt{m}$ selections
- ➔ For *n*-bit hash function, collision after ≈ $2^{n/2}$ random tests

Question: Given what you know about feasible/borderline/safe times for attacks, what digest size do you need to be safe against brute force against each property?

Attacks via Cryptanalysis

Idea: Use structure of hash function - don't just guess randomly!

Success of a cryptanalytic attack is measured by how much faster it is than brute force.

Good summary on Wikipedia "Hash function security summary" page:

	Preimage Resistance		Collision Resistance	
Algorithm	**Best Attack**	**Brute Force**	**Best Attack**	**Brute Force**
MD5	$2^{123.4}$	2^{128}	2^{18}	2^{64}
SHA-1	*No attack*	2^{160}	$2^{63.1}$	2^{80}
SHA-256	*No attack*	2^{256}	*No attack*	2^{128}

"*No attack*" means no attack is known that substantially improves upon brute force for the full-round version of the hash function.

Application 1: Password Storage

Problem: Need to store passwords in a database for checking logins

Goal: Passwords are checkable, but can't be stolen if DB compromised

Idea: Don't store *password* - store *H*(*password*)

What property of cryptographic hash functions must be satisfied?

Preimage resistance?

Second preimage resistance?

Collision resistance?

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Second preimage resistance? **No**

Collision resistance? **No**

Application 1: Password Storage

Additional issues with password storage:

Issue 1: Would be easy to make a dictionary of hashes of popular passwords.

Solution: Add "salt" - random values prepended to password before hashing

- Like an IV - must be stored with hash
- If set of salts is 10^{15} or larger, destroys possibility of dictionaries - see why?

Issue 2: Given salt and hash, can brute force password (hash fns are fast!)

Solution: Purposely slow down hash function by iterating

- Compute H(H(H(H(...H(salt+password)...))))
- Using SHA256, can hash around 10,000,000 passwords/second
- Iterate 1,000,000 times to slow down to 0.1 seconds per test

Question 1: How long to test 1,000,000 most common passwords with SHA256?

Question 2: What about with iterated SHA256?

Application 2: Detecting File Tampering

Problem: Detect if a file has been modified without a copy of original

Goal: Can check if file is the original from a "fingerprint"

Idea: Store *H*(*file*) as fingerprint - for any file, SHA256(*file*) just 32 bytes

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Preimage resistance? **No**

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Practical note:

Can't store hashes with files without additional protections!

Application 3: Verifying a message

Problem: I give you a contract, you verify what you agreed to with fingerprint of contract.

Example: Bank calls and asks "Did you agree to fingerprint xybqasd?"

Goal: I can't trick you into verifying a different contract than you saw

What property of cryptographic hash functions must be satisfied?

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Collision resistance? **Yes**

Practical note:

Seems esoteric, but this is precisely what happened when an MD5-based certification authority was compromised in 2008

Relation Between Different Properties

Some basic questions

- Does a function with collision resistance have second preimage resistance?
- Does a function with second preimage resistance have preimage resistance?
- Can you construct a function with preimage resistance but not collision resistance?

These questions will be explored in your next homework!

A sampling of other applications

Hash functions have been used for:

- Fast, secure pseudorandom number generation
- Disk deduplication
  - Similar: content-addressable storage as in Dropbox
- Forensic analysis (hashes of known files)
- Commitment protocols (commit to a value and reveal later)

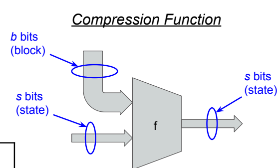
A new(-ish) application with a different property - proof of work

- Partial preimage: A preimage in which only part of the digest bits match
  - Example: Find SHA1 preimage in which first 40 bits of hash are 0
  - Should not be able to do this faster than 2^{40} tests on average
  - Smaller match requirement makes problem tractable - still hard though!
- Problem: Find x such that H(x || message) starts with b 0 bits
  - Invest time in finding x - so hard changing message requires similar time
  - Link to future messages - changing a past message now *very* expensive
  - This is the key concept behind Bitcoin mining and blockchain integrity

Classical hash function construction

Merkle-Damgard construction

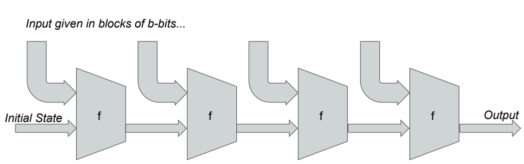
Used in MD5, SHA1, SHA256, SHA512, ...



Function	*b*	*s*
SHA1	512	160
SHA256	512	256
SHA512	1024	512

Classical hash function construction

Repeating compression function for long inputs



Notice that internal state is completely given in output if you stop early - this causes a problem with some later constructions, such as creating message authentication codes (MACs).

SHA-3

SHA-3 was selection process similar to that used for AES

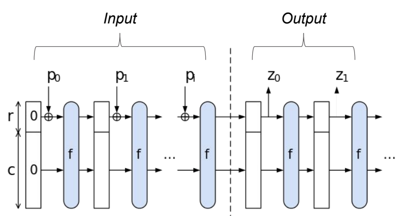
- Competition announced/started in 2006
- Context: Attacks had been made on MD4, SHA-0, and MD5, as well as on general structure - try to avoid “all designs alike”
  - From the competition announcement: “NIST also desires that the SHA-3 hash functions will be designed so that a possibly successful attack on the SHA-2 hash functions is unlikely to be applicable to SHA-3.”
- Selection after rounds of proposal/evaluate/narrow rounds
  - 51 submissions!
  - 14 hash functions selected for round 2 in 2009
  - 5 finalists selected in 2010
  - Winner was named Keccak - announced in 2012
    - Designed by Guido Bertoni, Joan Daemen, and Michaël Peeters, and Gilles Van Assche

Recognize this name?

SHA-3

Based on a “sponge function” (not Merkle-Damgard):

Input is “absorbed” into the sponge - output is “squeezed out”



Notice state include “unused capacity” bits (c) - can’t recover internal state to continue from output.