The University of North Carolina at Greensboro
CSC 653: Advanced Theory of Computation
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Handout 6
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Practice Problems 1

These problems are "practice problems" to prepare for the first exam. There are more problems here than will be on the exam, but they are approximately the same difficulty and depth of the problems that will be on the exam — as a result they are easier than many of the homework problems (where you have time to think about them), but they are also not just "memorize some facts to repeat" questions!

Students will work out and present solutions to as many of these problems as we can get to in class on Thursday, October 2, and we will discuss the solutions. Be prepared for that!

1. Consider the language $A = \{w \mid w$ contains no "run" of more than one consecutive 1$\}$, where A is over the alphabet $\{0, 1\}$. [Examples: 0010, 000, 001010 are all in A, but none of 011, 011000, 1110 are]

   (a) Draw a DFA that accepts A, and describe briefly how and why it works.

   (b) Give a regular expression for A, with a brief explanation.

2. Prove that $A = \{0^n 1^n \mid n \geq 1\}$ is not regular.

3. Prove that if $B = B^+$, then $BB \subseteq B$

4. Let N be an NFA with k states that recognizes a language A. Show that if A is non-empty then it must contain a string of length at most k.

5. Use the procedure given in the book for converting a regular expression into an NFA to produce an NFA for $(ab \cup a)^*$ — show your steps along the way to the final solution.

6. Give the formal definition of a pushdown automaton (you need to define everything with a sensible description, but you don't need to explain any of the definition).

7. Give a Context Free Grammar that generates $A = \{0^n 1^n 2^m 3^m \mid n \geq 1$ and $m \geq 1\}$.

8. Let $\Sigma = \{0, 1, 2, 3\}$ and $B = \{w \in \Sigma^* \mid w$ has an equal number of 0's and 1's, and an equal number of 2's and 3's$\}$. Show that B is not context-free.

9. Give a Context Free Grammar for $\{w\#x \mid w^R$ is a substring of $x\}$

10. Convert the following grammar into Chomsky Normal Form:

$$S \to aSb \mid \epsilon$$

11. State the pumping lemma for context free languages, and describe why it holds (I'm not asking for a formal proof, just a high-level description — a picture would probably be good to illustrate your description!)

12. Describe a Turing machine that decides $\{0^n1^n2^n \mid n \geq 1\}$ ("describe" doesn't mean you have to give an actual state transition function with all the details — but you do need to give a clear description of how it operates in terms of states and tape operatoins).

13. Show that if L is Turing-recognizable, then there is an enumerator for L.

14. Consider language $A = \{\langle G, w\rangle \mid G$ is a CNF grammar and w has more than one derivation in $G\}$. Show that A is Turing decidable.

15. Consider the language $A = \{\langle M\rangle \mid M$ is a DFA which doesn't accept any string with an odd number of 1's$\}$. Show that A is Turing decidable.

16. Show that the set of all infinite-length binary strings is uncountable.

17. Consider the following language: $C = \{\langle M, w\rangle \mid M$ is a Turing machine and never halts on input $w\}$. What can you say about the computability of C (is it Turing-decidable? Turing-recognizable? anything else you can say?). An informal explanation is fine here, but it should be clear and firmly grounded in reasoning about computability.